

Motion in Space

Kepler's Laws

Astronomer Nicolaus Copernicus published *On the Revolutions of the Heavenly Spheres* in 1543. Most people at that time thought that Earth was at the center of the universe. They also thought that the moon, planets, and stars rotated around Earth. But measurements always showed that the paths of the planets did not quite line up with this model. Copernicus said that Earth and other planets orbit the sun. He believed that these orbits were circular.

Kepler's three laws describe the motion of the planets.

Astronomer Tycho Brahe made many precise observations of the planets and stars. Some of Brahe's data did not agree with the model made by Copernicus. Astronomer Johannes Kepler worked for many years on this problem. Kepler's work led to three laws of planetary motion.

Remember

An ellipse is the shape traced by a point moving in a way that the sum of its distance from two points is constant. If the points are far apart, then the ellipse is long and narrow. If the points are on top of each other, then the ellipse becomes a circle.

Kepler's Laws of Planetary Motion	
First Law	Each planet travels in an elliptical orbit around the sun. The sun is at one of the focal points.
Second Law	An imaginary line drawn from the sun to any planet sweeps out equal areas in equal time intervals.
Third Law	The square of a planet's orbital period is proportional to the cube of its average distance from the sun. This can be written $T^2 \propto r^3$.

Kepler's first law states that the planets' orbits are ellipses. They are not circles. The second law states that planets move faster when they are close to the sun. They move slower when they are farther from the sun. The third law relates orbital period to orbital distance.

These rules apply to other orbiting objects too. These include the moon and satellites orbiting Earth.



READING CHECK

1. People once believed that the planets orbited Earth in perfect circles. What two changes happened to change this theory?

Kepler's laws are consistent with Newton's law of gravitation.

Kepler's laws were descriptive. Newton's theory of gravitation provided a force to explain them. This force is gravity. The fact that Kepler's laws closely matched observations gave more support for Newton's theory of gravitation.

Kepler's third law describes orbital period.

Think about an object orbiting something of mass m . The time for one orbit can be found with the following equation. The orbit's period is T .

$$T^2 = \left(\frac{4\pi^2}{Gm} \right) r^3$$

The period depends on the distance between the two and on the mass of the body being orbited. Note that it does not depend on the mass of the orbiting object.

The tangential speed v_t of an object in circular motion is equal to distance divided by time. This can be written $\frac{2\pi r}{T}$. Tangential speed is also called the *orbital speed*. The speed of an object in a circular orbit can then be calculated from the third law.

$$T = 2\pi \sqrt{\frac{r^3}{Gm}} \text{ and } v_t = \sqrt{\frac{Gm}{r}}$$

Many elliptical orbits can be shown as circles.

Did YOU Know?

We often think of the moon as orbiting Earth. But the moon and Earth are actually in orbit around a point between them. This point is inside Earth because Earth is much more massive than the moon is. The point would lie between them if they were the same size.



Critical Thinking

2. **Reasoning** What happens to the period of a satellite when the radius of its orbit increases?

Celestial Body Data							
Body	Mass (kg)	Mean radius (m)	Mean distance from sun (m)	Body	Mass (kg)	Mean radius (m)	Mean distance from sun (m)
Earth	5.97×10^{24}	6.38×10^6	1.50×10^{11}	Neptune	1.02×10^{26}	2.48×10^7	4.50×10^{12}
Earth's moon	7.35×10^{22}	1.74×10^6	—	Saturn	5.68×10^{26}	6.03×10^7	1.43×10^{12}
Jupiter	1.90×10^{27}	7.15×10^7	7.79×10^{11}	Sun	1.99×10^{30}	6.96×10^8	—
Mars	6.42×10^{23}	3.40×10^6	2.28×10^{11}	Uranus	8.68×10^{25}	2.56×10^7	2.87×10^{12}
Mercury	3.30×10^{23}	2.44×10^6	5.79×10^{10}	Venus	4.87×10^{24}	6.05×10^6	1.08×10^{11}

SAMPLE PROBLEM

A satellite orbits around Venus at a mean altitude of 3.61×10^5 m. What are its period and speed? Assume a circular orbit.

SOLUTION

1 ANALYZE

Determine what information is given and unknown.

Given: $h = 3.61 \times 10^5$ m (from the problem)

$$r_{\text{Venus}} = 6.05 \times 10^6 \text{ m and } m_{\text{Venus}} = 4.87 \times 10^{24} \text{ kg}$$

Unknown: $T = ?$ and $v_t = ?$

2 PLAN

Find the distance of the satellite from the center of Venus.

$$3.61 \times 10^5 \text{ m} + 6.05 \times 10^6 \text{ m} = 0.361 \times 10^6 \text{ m} + 6.05 \times 10^6 \text{ m}$$

$$r = 6.41 \times 10^6 \text{ m}$$

3 SOLVE

Use the equations for T and v_t .

$$\begin{aligned}
 T &= 2\pi \sqrt{\frac{r^3}{Gm}} = 2\pi \sqrt{\frac{(6.41 \times 10^6 \text{ m})^3}{(6.673 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2})(4.87 \times 10^{24} \text{ kg})}} \\
 &= 5.66 \times 10^3 \text{ s}
 \end{aligned}$$

$$\begin{aligned}
 v_t &= \sqrt{\frac{Gm}{r}} = \sqrt{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \frac{(4.87 \times 10^{24} \text{ kg})}{(6.41 \times 10^6 \text{ m})}} \\
 &= 7.12 \times 10^3 \text{ m/s}
 \end{aligned}$$

SAMPLE PROBLEM (continued)

4 CHECK YOUR WORK

Does $2\pi r = v_t T$? Does the length of one orbit equal the orbital speed multiplied by the time for one orbit?

$$v_t T = (7.12 \times 10^3 \text{ m/s})(5.66 \times 10^3 \text{ s}) = 4.03 \times 10^7 \text{ m}$$

$$2\pi r = 2\pi(6.41 \times 10^6 \text{ m}) = 4.03 \times 10^7 \text{ m}$$

PRACTICE

- A. At what distance above Earth would a satellite have a period of 125 minutes? This can also be written as 7500 s.

Weight and Weightlessness

Weight is the magnitude of the force due to gravity. Think about what happens when you step on a scale. The scale measures the downward force exerted on it. The scale will measure a force greater than your weight if someone else pushes down on the scale at the same time. The scale will measure a force less than your weight if you rest part of your weight on a counter or the floor. The scale reading is the net result of your weight and any other force that is applied.

Imagine you are standing in an elevator. You feel lighter when it starts to move downward. This is due to your inertia resisting the change in your position. A scale would show your weight decreasing as you accelerate. Your weight returns to normal once the elevator and you moved together at constant speed.

You feel heavier when the elevator stops. This is due to your inertia resisting this change in your motion. A scale would show your weight increasing as you accelerated. Your weight returns to normal once the elevator and you were both at a stop.

The scale is recording the normal force it exerts to hold you up. This force is greater when the scale accelerates toward you. This force is smaller when the scale accelerates away from you.



Critical Thinking

3. **Apply Concepts** Suppose someone weighs 625 N. He stands on a scale. Someone helps support him with an upward force of 225 N. What value will the scale show in newtons?

Suppose you and the scale were both falling freely. The scale would not exert any normal force to hold you up. It would read zero.

Astronauts in orbit experience apparent weightlessness.

Astronauts floating in the International Space Station experience *apparent weightlessness*. They still have mass and it is still attracted to Earth's mass. But the space station and everything in it is falling at the same rate. Recall the illustration of a cannonball falling in a curved path that matches the curve of Earth. Weightlessness occurs only in deep space. But even in deep space there is some gravitational attraction between objects that are far away from each other.

SECTION 7.3 REVIEW

REVIEWING MAIN IDEAS

1. Explain how Kepler's laws of planetary motion relate to Newton's law of universal gravitation.

2. Why are you lighter on the moon than on Earth?

Critical Thinking

3. A roller coaster raises people high into the air. Then it drops them at the rate of free-fall acceleration. Are the people experiencing apparent weightlessness? True weightlessness? Neither? Explain.